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COUPLED AT BOUNDARY MASS OR HEAT TRANSFER IN ENTRANCE CONCURRENT FLOW

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Abstract—The analytical solutions for coupled at boundary diffusion processes in the entrance region of channel flow are presented. An algorithm for computing the resulting eigenvalue problem and computer program CUPFLOW are developed for studying a class of coupled problems to which belong the ones discussed in [1–7]. Some illustrative examples are also included.

NOMENCLATURE

$L_{n1}, L_{n2}, L_{n3}, L_{n4}, f_n, \omega$,	prescribed constants;
$k_n(\xi), w_n(\xi)$,	prescribed functions;
i,	$1, 2, 3, \ldots, \infty$;
n,	1 or 2;
ξ, η,	dimensionless radial and axial distances;
$\theta_n(\xi, \eta)$,	dimensionless potentials;
μ_i,	eigenvalues;
$\psi_{ni}(\xi)$,	eigenfunctions;
Γ_n,	factor of the form;
$W_{\Gamma_n}(\xi), V_{\Gamma_n}(\xi)$,	functions defined by equations (27) and (28).

1. INTRODUCTION

THE DESIGN of a separating process is usually based on theoretical predictions, which are derived from the analytical mass transfer solutions. The distributions of concentrations in gas–liquid or liquid–liquid flows have been found in [1–4]. The mass transfer from one medium to another leads to the pure diffusion equations coupled only at boundary conditions.

Similar problems appear in the study of the heat-transfer coefficients in concurrent flow double pipe heat exchangers [5, 6], simultaneous heat and mass transfer in internal gasflows in a duct whose walls are coated with a sublimable material [7] and elsewhere.

In a recent paper [8], one of the authors presented the general solution of the diffusion equations coupled through general boundary conditions, including the problems in the reference mentioned so far as very special cases. The main difficulty in the application of the solutions given in [8] is the tackling of the resulting eigenvalue problem, which is not of the conventional type.

In this paper is described the algorithm for computing two Sturm–Liouville equations coupled at the common boundary. Using this algorithm a computer program CUPFLOW was compiled which permits the study of a class of coupled problems to which belong the ones discussed in [1–7].

2. ANALYSIS

The differential equations governing the mass transfer in an entrance concurrent flow may be written in a general form

$$\omega^{2(n-1)}w_n(\xi)\xi^{\Gamma_n}\frac{\partial\theta_n(\xi,\eta)}{\partial\eta} = \frac{\partial}{\partial\xi}\left\{\xi^{\Gamma_n}k_n(\xi)\frac{\theta_n(\xi,\eta)}{\partial\xi}\right\} \quad (1)$$

for $n = 1, 2$; $\Gamma_n = 0$ or 1; $0 \leqslant \xi \leqslant 1$; $\eta \geqslant 0$ and $k_n(1) = 1$.

The initial and boundary conditions are given by

$$\theta_n(\xi, 0) = f_n \quad (2)$$

$$\frac{\partial\theta_n(0,\eta)}{\partial\xi} = 0 \quad (3)$$

$$L_{n1}\theta_1(1,\eta) + L_{n2}\theta_2(1,\eta) + L_{n3}\frac{\partial\theta_1(1,\eta)}{\partial\xi} + L_{n4}\frac{\partial\theta_2(1,\eta)}{\partial\xi} = 0 \quad (4)$$

where $L_{11} = L_{12} = 0$ and $L_{13}L_{14}L_{21}L_{22} \geqslant 0$.

In equation (1) $w_n(\xi)$ are the normalized velocities in direction therefore we have

$$(\Gamma_n+1)\int_0^1 \xi^{\Gamma_n}w_n(\xi)\,\mathrm{d}\xi = 1. \quad (5)$$

The solutions of equations (1)–(4) can be easily obtained as a very special case from the general theory presented in [8]. The results are

$$\theta_n(\xi,\eta) = (-1)^{n+1}L_{2,3-n} \times \frac{(\Gamma_2+1)L_{13}f_1 + (\Gamma_1+1)L_{14}f_2\omega^2}{(\Gamma_2+1)L_{13}L_{22} - (\Gamma_1+1)L_{14}L_{21}\omega^2} - \sum_{i=1}^{\infty} \times \frac{\sum_{n=1}^{2}\sigma_n f_n \psi'_{ni}(1)}{\sum_{n=1}^{2}\sigma_n\left\{\left(\frac{\partial\psi_n(1)}{\partial\mu}\right)_{\mu=\mu_i}\psi'_{ni}(1) - \left(\frac{\partial^2\psi_n(1)}{\partial\xi\partial\mu}\right)_{\mu=\mu_i}\psi_{ni}(1)\right\}} \times \frac{2}{\mu_i}\psi_{ni}(\xi)\mathrm{e}^{-\mu_i^2\eta} \quad (6)$$

where

$$\sigma_1 = L_{13}L_{21}, \quad \sigma_2 = -L_{14}L_{22}$$

and μ_i are eigenvalues of the two-region Sturm–Liouville problem

$$\{\xi^{\Gamma_n}k_n(\xi)\psi'_{ni}(\xi)\}' + \mu_i^2\omega^{2(n-1)}\xi^{\Gamma_n}w_n(\xi)\psi_{ni}(\xi) = 0 \quad (7)$$

$$\psi'_{ni}(0) = 0 \quad (8)$$

$$L_{n1}\psi_{1i}(1) + L_{n2}\psi_{2i}(1) + L_{n3}\psi'_{1i}(1) + L_{n4}\psi'_{2i}(1) = 0. \quad (9)$$

Equations (6) are derived from the general solutions given in [8] assuming that

$$\left(\frac{\partial\psi_n(0)}{\partial\mu}\right)_{\mu=\mu_i} = 0, \quad \left(\frac{\partial^2\psi_n(0)}{\partial\xi\partial\mu}\right)_{\mu=\mu_i} = 0. \quad (10)$$

Of importance to the application of solutions (6) is the numerical calculation of the eigenvalues μ_i and eigenfunctions $\psi_{ni}(\xi)$ from (7) to (9). In order to find them it is appropriate to introduce new variables $y_{4n-2}(\xi)$ and $y_{4n-3}(\xi)$. Through the substitutions

$$\psi_{ni}(\xi) = A_n y_{4n-2,i}(\xi), \quad \xi^{\Gamma_n}k_n(\xi)\psi'_{ni}(\xi) = A_n y_{4n-3,i}(\xi) \quad (11)$$

the system (7) is reduced to

$$y'_{4n-3,i}(\xi) = -\mu_i^2\omega^{2(n-1)}\xi^{\Gamma_n}w_n(\xi)y_{4n-2,i}(\xi) \quad (12)$$

$$y'_{4n-2,i}(\xi) = y_{4n-3,i}(\xi)/(\xi^{\Gamma_n}k_n(\xi)). \quad (13)$$

These equations (12)–(13) will be integrated numerically with conditions

$$y_{4n-3,i}(0) = 0, \quad y_{4n-2,i}(0) = 1 \quad (14)$$

and therefore conditions (8) are automatically satisfied.

Substituting (12) and (13) in boundary conditions (9) we come to the expressions

$$A_1L_{13}y_{1i}(1) + A_2L_{14}y_{5i}(1) = 0 \quad (15)$$

$$A_1\{L_{21}y_{2i}(1) + L_{23}y_{1i}(1)\} + A_2\{L_{22}y_{1i}(1) + L_{24}y_{5i}(1)\} = 0. \quad (16)$$

The systems obtained are homogeneous; then it follows

$$L_{13}L_{22}y_{6i}(1)/y_{5i}(1) - L_{14}L_{21}y_{2i}(1)/y_{1i}(1) + L_{13}L_{24} - L_{14}L_{23} = 0. \quad (17)$$

Further we define the variables

$$y_{4n,i}(\xi) = \frac{\partial y_{4n-2,i}(\xi)}{\partial\mu}, \quad y_{4n-1,i}(\xi) = \frac{\partial y_{4n-3,i}(\xi)}{\partial\mu}. \quad (18)$$

Now equations (12) and (13) after differentiation in μ become

$$y'_{4n-1,i}(\xi) = -\mu_i\omega^{2(n-1)}\xi^{\Gamma_n}w_n(\xi) \times \{\mu_i y_{4n,i}(\xi) + 2y_{4n-2,i}(\xi)\} \quad (19)$$

$$y'_{4n,i}(\xi) = y_{4n-1,i}(\xi)/(\xi^{\Gamma_n}k_n(\xi)). \quad (20)$$

Equations (19)–(20) will be integrated numerically at conditions

$$y_{4n-1,i}(0) = 0, \quad y_{4n,i}(0) = 0 \quad (21)$$

so as to satisfy equations (10). Thus we have

$$\left(\frac{\partial\psi_n(1)}{\partial\mu}\right)_{\mu=\mu_i} = A_n y_{4n,i}(1)$$
$$\left(\frac{\partial^2\psi_n(1)}{\partial\xi\partial\mu}\right)_{\mu=\mu_i} = A_n y_{4n-1,i}(1). \quad (22)$$

Using equations (11), (18) and standard mathematical techniques the solutions (6) may be rearranged as

$$\theta_n(\xi,\eta) = (-1)^{n+1}L_{2,3-n} \times \frac{(\Gamma_2+1)L_{13}f_1 + (\Gamma_1+1)L_{14}f_2\omega^2}{(\Gamma_2+1)L_{13}L_{22} - (\Gamma_1+1)L_{14}L_{21}\omega^2} + (-1)^nL_{1,5-n}(L_{21}f_1 + L_{22}f_2) \times \sum_{i=1}^{\infty} D_i y_{9-4n,i}(1)y_{4n-2,i}(\xi)e^{-\mu_i^2\eta} \quad (23)$$

where

$$D_i = \frac{2}{\mu_i}y_{1i}(1)y_{5i}(1)\left\{L_{14}L_{21}y_{5i}^2(1)\begin{vmatrix} y_{4i}(1) & y_{3i}(1) \\ y_{2i}(1) & y_{1i}(1)\end{vmatrix} - L_{13}L_{22}y_{1i}^2(1)\begin{vmatrix} y_{8i}(1) & y_{7i}(1) \\ y_{6i}(1) & y_{5i}(1)\end{vmatrix}\right\}^{-1} \quad (24)$$

If we use an approximation in which the actual velocity profile is replaced by the average (plug flow case), then $w_n(\xi) = 1$ and systems (12)–(13) have exact analytical solutions

$$y_{4n-2,i}(\xi) = W_{\Gamma_n}(\mu_i\omega^{n-1}\xi) \quad (25)$$

$$y_{4n-3,i}(\xi) = -\mu_i\omega^{n-1}\xi^{\Gamma_n}V_{\Gamma_n}(\mu_i\omega^{n-1}\xi). \quad (26)$$

The properties of the functions

$$W_{\Gamma_n}(\xi) = \sum_{j=0}^{\infty}\frac{(-1)^j\xi^{2j}}{(2j)!!(\Gamma_n+2j-1)!!} \quad (27)$$

$$V_{\Gamma_n}(\xi) = \sum_{j=0}^{\infty}\frac{(-1)^j\xi^{2j+1}}{(2j)!!(\Gamma_n+2j+1)!!} \quad (28)$$

are described in detail in the monograph [9] and partially in the Appendix of [10]. For $\Gamma_n = 0$ or 1 the series (27) and (28) define the following well-known functions.

$$W_0(x) = \cos x, \quad W_1(x) = J_0(x),$$
$$V_0(x) = \sin x, \quad V_1(x) = J_1(x). \quad (29)$$

Using (25) and (26) equations (17) are reduced to

$$L_{14}L_{21}W_{\Gamma_1}(\mu)/V_{\Gamma_1}(\mu) - L_{13}L_{22}W_{\Gamma_2}(\mu\omega)/[\omega V_{\Gamma_2}(\mu\omega)] + \mu(L_{13}L_{24} - L_{14}L_{23}) = 0. \quad (30)$$

The positive nonzero roots of equation (30) give the eigenvalues μ_i.

Introducing into solutions (23) the new $y(\xi)$ variables we obtain

$$\theta_n(\xi,\eta) = (-1)^{n+1}L_{2,3-n} \times \frac{(\Gamma_2+1)L_{13}f_1 + (\Gamma_1+1)L_{14}f_2\omega^2}{(\Gamma_2+1)L_{13}L_{22} - (\Gamma_1+1)L_{14}L_{21}\omega^2} + (-1)^{n+1}L_{1,5-n}(L_{21}f_1 + L_{22}f_2) \times \sum_{i=1}^{\infty} D_i\mu_i\omega^{2-n}V_{\Gamma_{3-n}}(\mu_i\omega^{2-n}) \times W_{\Gamma_n}(\mu_i\omega^{n-1}\xi)e^{-\mu_i^2\eta} \quad (31)$$

where

$$D_i = 2\frac{V_{\Gamma_1}(\mu_i)}{\mu_i}\frac{V_{\Gamma_2}(\mu_i\omega)}{\mu_i\omega}\left\{L_{14}L_{21}V_{\Gamma_2}^2(\mu_i\omega)\right.$$
$$\times\left[W_{\Gamma_1}^2(\mu_i)+V_{\Gamma_1}^2(\mu_i)+(1-\Gamma_1)W_{\Gamma_1}(\mu_i)\frac{V_{\Gamma_1}(\mu_i)}{\mu_i}\right]\Big\}$$
$$-L_{13}L_{22}V_{\Gamma_1}^2(\mu_i)\left[W_{\Gamma_2}^2(\mu_i\omega)+V_{\Gamma_2}^2(\mu_i\omega)\right.$$
$$\left.\left.+(1-\Gamma_2)W_{\Gamma_2}(\mu_i\omega)\frac{V_{\Gamma_2}(\mu_i\omega)}{\mu_i\omega}\right]\right\}. \quad (32)$$

3. ALGORITHM FOR SOLVING THE PROBLEM

The determination of the potentials $\theta_n(\xi,\eta)$ from equation (31) is easily performed if the roots of the transcendental equation (30) are calculated. Similarly, for the solution of (23) it is necessary to calculate the eigenvalues μ_i through numerical integration of the system (12)–(13) with the conditions (14) so that condition (17) is satisfied.

The calculation of μ_i is considerably facilitated if the intervals, where the eigenvalues are to be found, are known. Let us consider equation (30) for the case of parallel planes $\Gamma_1 = \Gamma_2 = 0$, and $L_{13}L_{22} = 1$, $L_{14}L_{21} = 1$, $L_{13}L_{24} - L_{14}L_{23} = 0.5$, $\omega = 1.6$. Figure 1 shows the functions $z_1 = \cot\text{an}\,\mu$, $z_2 = -(1/\omega)\cot\text{an}\,\omega\mu$ and $z_3 = 0.5\,\mu$. The roots of equation (30) are determined from the points of intersection of the functions z_1+z_3 and z_2. When $L_{14}L_{21}$ and $L_{13}L_{22}$ are of the same signs, even if their values are different from 1, the roots of equation (30) lie between two subsequent roots of the equations

$$\sin\mu = 0 \quad \text{and} \quad \sin(\omega\mu) = 0. \quad (33)$$

We have to note that if for some value of μ the roots of equation (33) incidentally coincide then this value of μ is a root of equation (30) too.

Having in mind that the behavior of the functions $\mu y_{2i}(1)/y_{1i}(1)$ and $\mu y_{6i}(1)/y_{5i}(1)$ is similar to that of the $\cot\text{an}\,\mu$ we can draw the conclusion that every root of equation (17) lies between two subsequent roots of the equations:

$$y_{1i}(1) = 0 \quad \text{and} \quad y_{5i}(1) = 0. \quad (34)$$

It is obvious that equations (34) define the eigenvalues of the classical Sturm–Liouville equation; asymptotical formulae are derived for them in [11].

On the basis of the consideration given above the following algorithm was successfully applied for the calculation of the eigenvalues of equations (12)–(13) with the conditions (14)–(17):

1. Calculate the eigenvalues μ_i^* by numerical integration (Runge–Kutta method) of equations (12)–(13) for $n = 1$ with the boundary condition (14), so that with Newton's iterative method the equation $y_{1i}(1) = 0$ also be satisfied. The calculation is started with the initial approximation

$$\mu_i^* = \pi\left(i+\frac{1+3\Gamma_1}{12}\right)\Big/\int_0^1 \sqrt{[w_1(\xi)/k_1(\xi)]}\,d\xi \quad (35)$$

and 2 iterations are preformed.

2. Calculate in the same way the eigenvalues μ_i^{**} solving numerically equations (12)–(13) for $n = 2$ at the condition (14), so that the equation $y_{5i}(1) = 0$ be satisfied. The calculation is started with

$$\mu_i^{**} = \frac{\pi}{\omega}\left(i+\frac{1+3\Gamma_2}{12}\right)\Big/\int_0^1 \sqrt{[w_2(\xi)/k_2(\xi)]}\,d\xi \quad (36)$$

and also 2 iterations are performed.

3. The eigenvalues μ_i^* and μ_i^{**} so obtained are arranged according to their values and in this way the intervals, where the eigenvalues μ_i of equation (17) lie, are determined.

4. The eigenvalues μ_i and eigenfunctions $y_{4n-2,i}(\xi)$ and $y_{4n-3,i}(\xi)$ are determined through direct numerical solution of equations (12)–(13) at condition (14) using the method of successive bisection, so that equation (17) is identically satisfied.

5. The functions $y_{4n,i}(\xi)$ and $y_{4n-1,i}(\xi)$ are calculated numerically from equations (19)–(20) at conditions (21).

6. Using formulae (23) and (24) $\theta_n(\xi,\eta)$ are calculated.

On the basis of this algorithm a computer program CUPFLOW was compiled and the examples considered in [1–7] were recalculated.

4. ILLUSTRATIVE RESULTS

As a first example let us consider the hydrodynamically developed flow of a gas in a duct whose walls are coated with a sublimable material. The entering gas flow contains vapours of the sublimable material in the amount less than the saturation value corresponding to the temperature and the total pressure. In addition the latent heat of sublimation is supplied to the wall by the gas itself. The full formulation of the problem is given in [7] and leads to the equations:

$$u(\xi)\frac{\partial\theta(\xi,\eta)}{\partial\eta} = \frac{\partial^2\theta(\xi,\eta)}{\partial\xi^2}, \quad Lu(\xi)\frac{\partial\varphi(\xi,\eta)}{\partial\eta} = \frac{\partial^2\varphi(\xi,\eta)}{\partial\xi^2} \quad (37)$$

$$\theta(\xi,0) = 1, \quad \varphi(\xi,0) = 1 \quad (38)$$

$$\frac{\partial\theta(0,\eta)}{\partial\xi} = 0, \quad \frac{\partial\varphi(0,\eta)}{\partial\xi} = 0 \quad (39)$$

$$L\frac{\partial\theta(1,\eta)}{\partial\xi} - \frac{\partial\varphi(1,\eta)}{\partial\xi} = 0, \quad K\theta(1,\eta)+\varphi(1,\eta) = 0 \quad (40)$$

where: $u(\xi)$ = dimensionless velocity, θ = dimensionless temperature, φ = mass fraction variable, L and $K = \alpha\lambda_s/c_p$ are dimensionless numbers defined in [7].

This problem is solved in [7] for $u(\xi) = 1$. To obtain the solution from equations (37)–(40) it is sufficient to let: $\omega^2 = L$, $\Gamma_1 = \Gamma_2 = 0$, $k_1(\xi) = k_2(\xi) = 1$, $f_1 = f_2 = 1$, $L_{13} = L$, $L_{14} = -1$, $L_{21} = K$, $L_{22} = 1$, $L_{23} = L_{24} = 0$.

The distribution of the temperature and the mass fraction are calculated using 10 eigenvalues. In Fig. 2 are plotted some typical results for: $L = 0.81$, $K = 0.1$ and $u(\xi) = \frac{3}{2}(1-\xi^2)$.

As a second example we consider concurrent mass transfer between fluid and gas flowing separately in a

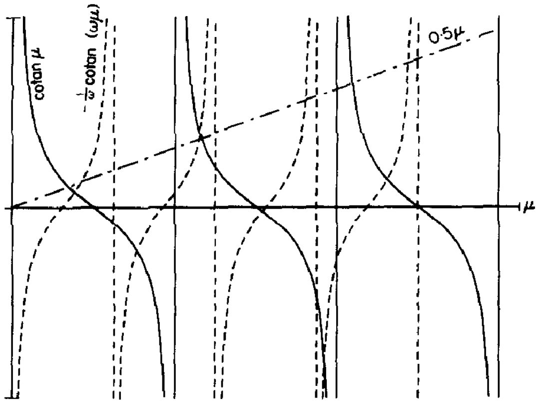

FIG. 1.

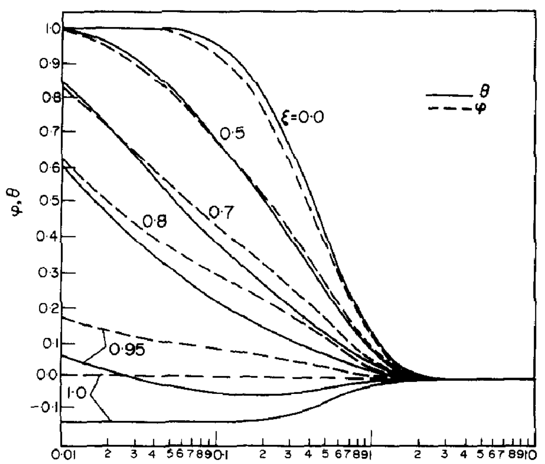

FIG. 2.

parallel-plate channel. This problem is defined in [3, 4] through the following equations:

$$\beta^2 u_1(\xi)\frac{\partial c_1(\xi,\eta)}{\partial \eta} = \frac{\partial^2 c_1(\xi,\eta)}{\partial \xi^2},$$
$$u_2(\xi)\frac{\partial c_2(\xi,\eta)}{\partial \eta} = \frac{\partial^2 c_2(\xi,\eta)}{\partial \xi^2} \quad (41)$$

$$c_1(\xi,0) = 1, \quad c_2(\xi,0) = 0 \quad (42)$$

$$\frac{\partial c_1(0,\eta)}{\partial \xi} = 0, \quad \frac{\partial c_2(0,\eta)}{\partial \xi} = 0 \quad (43)$$

$$\frac{\partial c_1(1,\eta)}{\partial \xi} + \beta^2 \varepsilon \frac{\partial c_2(1,\eta)}{\partial \xi} = 0, \quad c_1(1,\eta) - c_2(1,\eta) = 0 \quad (44)$$

where the dimensionless parameters are: c_1 = concentration in gas flow, c_2 = concentration in the liquid, β^2 and ε are numbers defined in [3, 4].

Dimensionless velocities for the laminar flow case are [3, 4]:

$$u_1(\xi) = \tfrac{3}{2}(1-\xi^2), \quad u_2(\xi) = \tfrac{3}{2}\xi(1-\xi). \quad (45)$$

The problem defined by equations (41)–(45) is also a special case of the problem studied here: $\Gamma_1 = \Gamma_2 = 0$, $k_1(\xi) = k_2(\xi) = 1$, $\omega^2 = \beta^2$, $f_1 = 0$, $f_2 = 1$, $L_{13} = \beta^2\varepsilon$, $L_{14} = 1$, $L_{21} = -1$, $L_{22} = 1$, $L_{23} = L_{24} = 0$. In Fig. 3 is plotted one of the examples calculated with $\varepsilon = 0.5$ and $\beta^2 = 0.2$.

As a last example let us consider the heat transfer

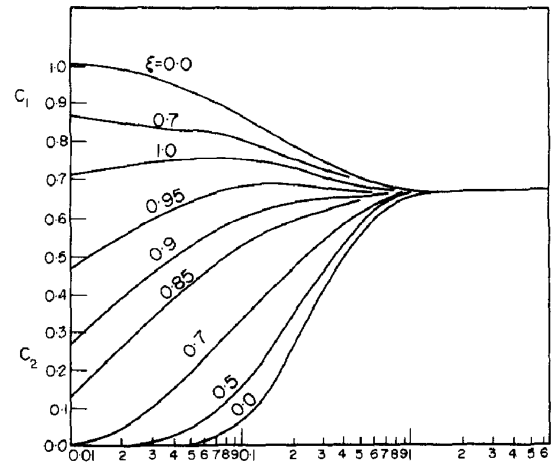

FIG. 3.

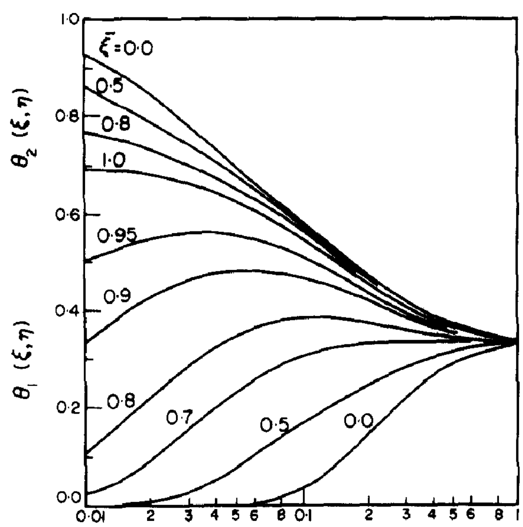

FIG. 4.

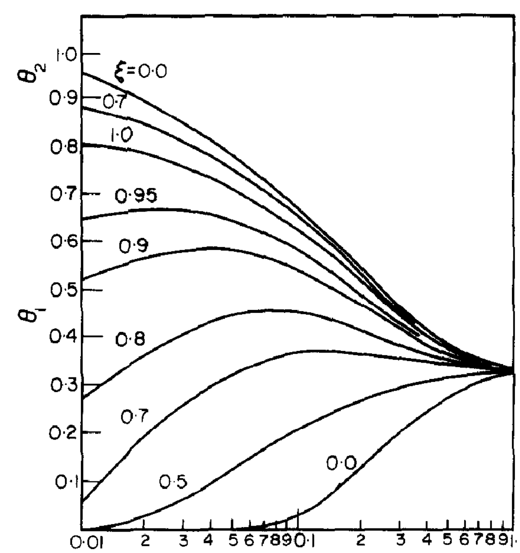

FIG. 5.

in an exchanger which consists of two concentric circular pipes with fluids concurrently flowing through the annular space and the central tube. The fluids enter with different temperatures, exchanging heat through the common wall. This problem is studied in [5, 6] and leads to

$$\xi^{\Gamma}u_1(\xi)\frac{\partial\theta_1(\xi,\eta)}{\partial\eta}=\frac{\partial}{\partial\xi}\left[\xi^{\Gamma}\frac{\partial\theta_1(\xi,\eta)}{\partial\xi}\right],$$
$$\frac{KH}{\Gamma+1}u_2(\xi)\cdot\frac{\partial\theta_2(\xi,\eta)}{\partial\eta}=\frac{\partial^2\theta_2(\xi,\eta)}{\partial\xi^2} \tag{46}$$

$$\theta_1(\xi,0)=0,\quad \theta_2(\xi,0)=1 \tag{47}$$

$$\frac{\partial\theta_1(0,\eta)}{\partial\xi}=0,\quad \frac{\partial\theta_2(0,\eta)}{\partial\xi}=0 \tag{48}$$

$$K\frac{\partial\theta_1(1,\eta)}{\partial\xi}+\frac{\partial\theta_2(1,\eta)}{\partial\xi}=0,$$
$$K_w\frac{\partial\theta_1(\xi,\eta)}{\partial\xi}+\theta_1(1,\eta)-\theta_2(1,\eta)=0 \tag{49}$$

where: θ_1 and θ_2 are the temperatures in the central tube and the annular space respectively, $\Gamma = 0$ or 1 for a parallel plane or a tube exchanger, K, H and K_w are dimensionless numbers defined in [5, 6] and called fluid thermal resistance ratio, heat capacity flow rate ratio and wall thermal resistance ratio respectively.

To obtain the solution it is necessary to assume that: $k_1(\xi) = k_2(\xi) = 1$, $f_1 = 0$, $f_2 = 1$, $\omega^2 = (KH/\Gamma+1)$, $\Gamma_1 = \Gamma$, $\Gamma_2 = 0$, $L_{13} = K$, $L_{14} = 1$, $L_{21} = 1$, $L_{23} = -1$, $L_{23} = K_w$, $L_{24} = 0$.

In Fig. 4 are shown the temperature distribution for $K = 0.1$, $H = 0.5$, $K_w = 0$, $\Gamma = 0$ and the plug flow case: $u_1(\xi) = u_2(\xi) = 1$.

In the last Fig. 5 is presented the same example for the laminar flow case: $u_1(\xi) = 2(1-\xi^2)$ and $u_2(\xi) = 6\xi(1-\xi)$.

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TRANSFERT DE CHALEUR ET DE MASSE COUPLES A LA FRONTIERE DANS LA REGION D'ETABLISSEMENT D'UN ECOULEMENT PARALLELE

Résumé—On présente des solutions analytiques des processus de diffusion couplés aux frontières dans la région d'établissement d'un écoulement en canal. Un algorithme est développé pour résoudre le problème de valeurs propres résultant, ainsi qu'une procédure numérique CUPFLOW, afin d'étudier une classe de problèmes couplés à laquelle appartiennent ceux discutés dans [1–7]. Quelques exemples sont aussi présentés.

GEKOPPELTER WÄRME- UND STOFFÜBERGANG IM EINLAUFBEREICH GLEICHGERICHTETER STRÖMUNGEN

Zusammenfassung—Es werden analytische Lösungen angegeben für Diffusionsprozesse in hydrodynamisch nicht ausgebildeter Kanalströmung mit Kopplung im Grenzbereich. Zur Berechnung der resultierenden Eigenwerte ist ein Algorithmus und ein Computerprogramm CUPFLOW angegeben. Eine Reihe von Problemen der genannten Art aus der Literatur wurden untersucht. Anschauliche Beispiele werden ebenfalls wiedergegeben.

СОВМЕСТНЫЙ МАССО- ИЛИ ТЕПЛООБМЕН НА ГРАНИЦЕ ПРИ СПУТНОМ ТЕЧЕНИИ НА ВХОДЕ В КАНАЛ

Аннотация — В докладе представлены аналитические решения совместных диффузионных процессов на начальном участке канала. Для исследования класса совместных задач (к ним относится задача, рассматриваемая в [1–7]), разработаны алгорифм для вычисления результирующих задач о собственных значениях и программа «CUPFLOW» для вычислительной машины. В докладе содержится несколько пояснительных примеров.